Center for Biomedical Imaging

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STABILITY OF IMAGE-RECONSTRUCTION ALGORITHMS

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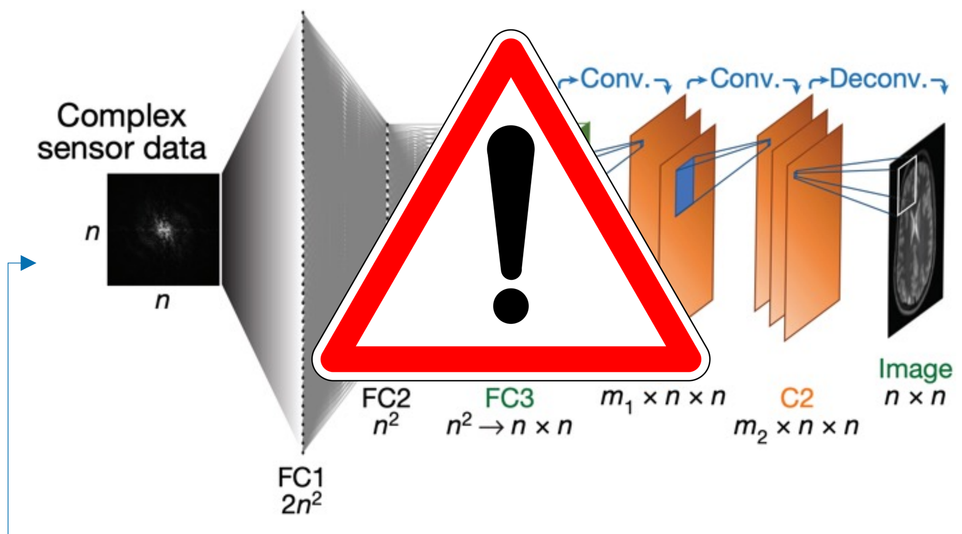
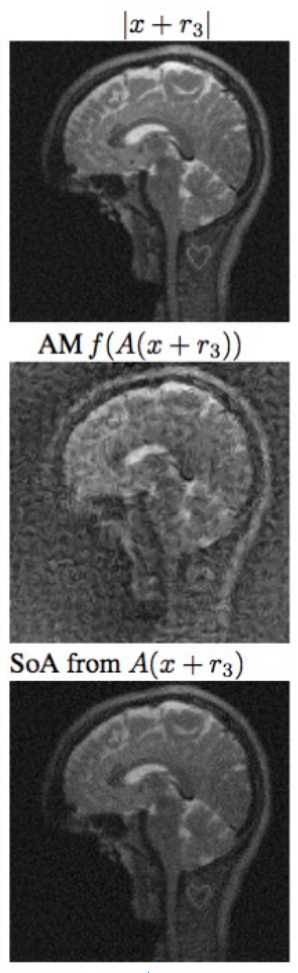
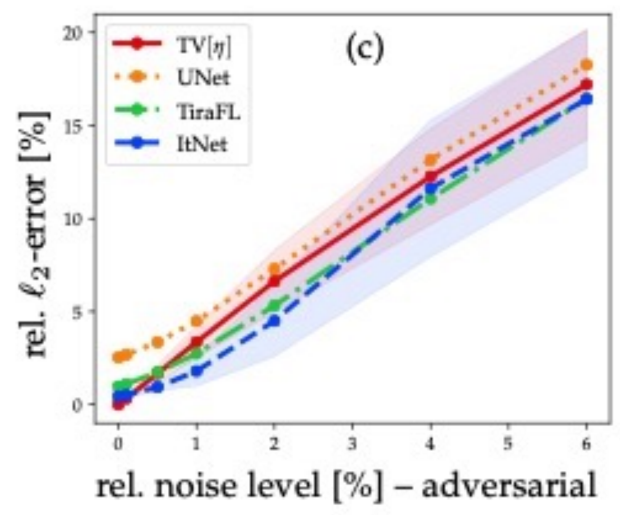
BACKGROUND



Zhu, B., Liu, J., Cauley, S. et al. "Image reconstruction by domain-transform manifold learning," Nature 555, 487–492 (2018).



V. Antun, F. Renna, C. Poon, B. Adcock, and A.C. Hansen, "On instabilities of deep learning in image reconstruction and the potential costs of AI," Proceedings of the National Academy of Sciences 48 (117), pp. 30088–30095, 2020



M. Genzel, J. Macdonald, and M. Marz "Solving inverse problems with deep neural networks - Robustness included," IEEE Transactions on Pattern Analysis and Machine Learning - Early Access (2022)

- Dangerous NN-based image-reconstruction methods (*e.g.*, Zhu 2018).
- Stability problems (*e.g.*, Antun 2020).
- Questioning traditional methods (*e.g.*, Genzel 2022).

AIMS

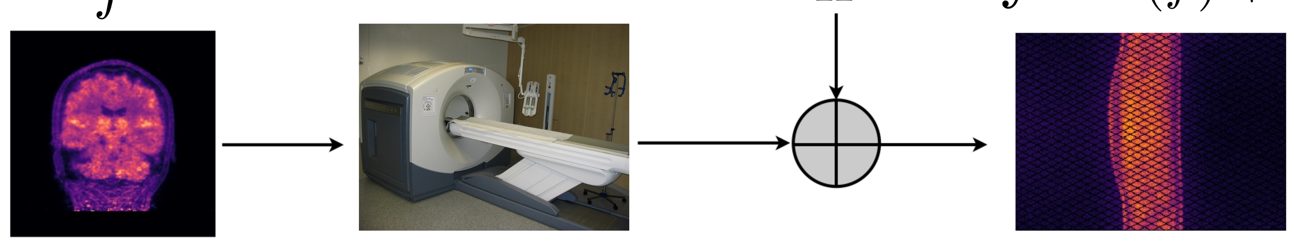
- Towards the objective, quantitative comparison of stability in image-reconstruction methods.
- Obtain (computable) bounds of the form:

$$\|\mathbf{f}_{\mathbf{y}_1} - \mathbf{f}_{\mathbf{y}_2}\|_{\ell_p} \leq K(Y)\,\|\mathbf{y}_1 - \mathbf{y}_2\|_2^{\beta},$$

where $\mathbf{f}_{\mathbf{y}}$ is the reconstruction for a measurement vector $\mathbf{y} \in \mathbb{R}^M$

- Focus on ℓ_p-regularized linear inverse problems

$$\min_{f \in \ell_p} \left\{ E\left(\mathbf{y}, \boldsymbol{\nu}(f)\right) + \lambda \|f\|_{\ell_p}^p \right\}$$



METHODS

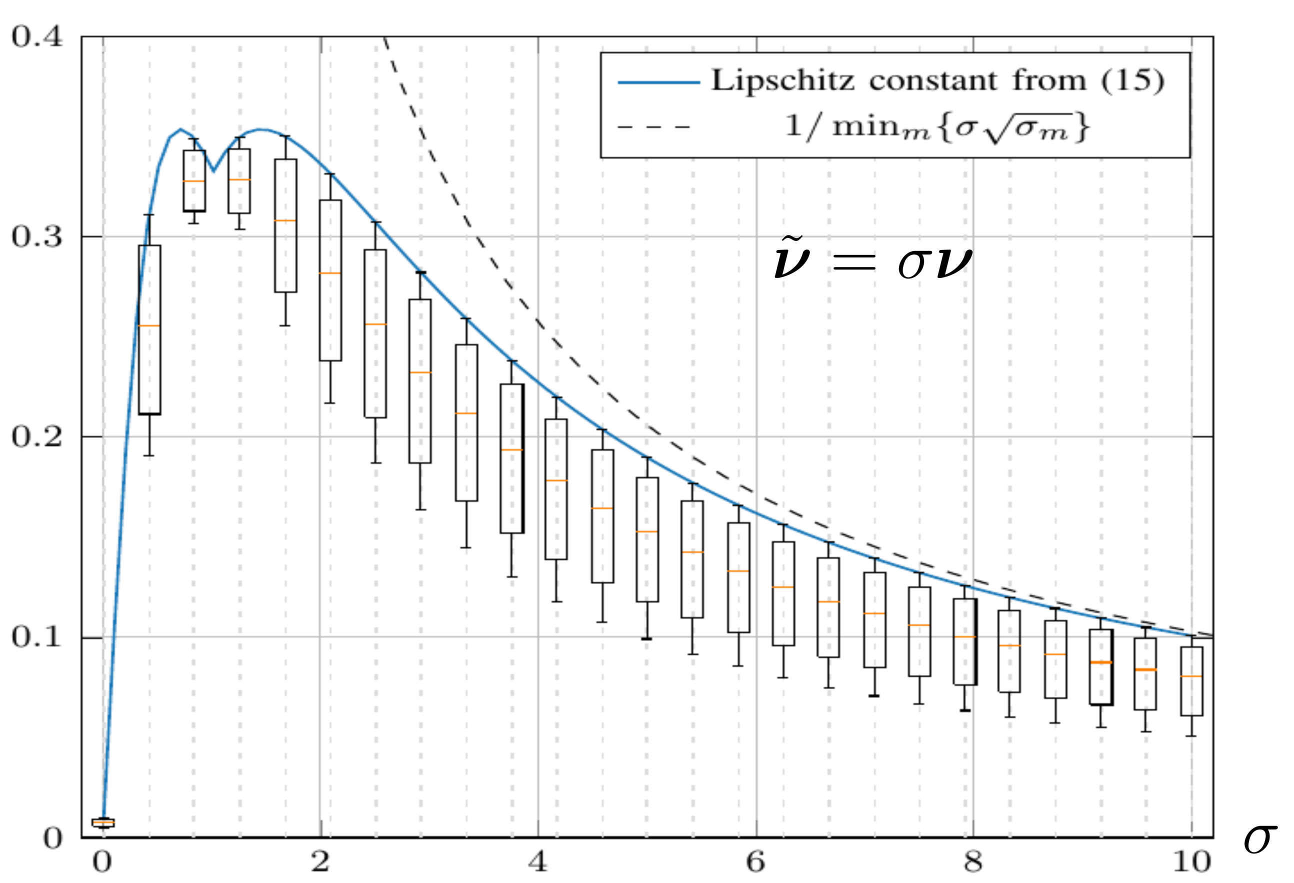
- Analysis of sufficient conditions for stability in abstract optimization problems that depend on the measurements and the reconstructed image.
  - Gradient of the data-fidelity term w.r.t. f has to be Lipschitz w.r.t. y
  - Regularizer has to be α-uniform convex
- Analysis of the growth of the gradient of the ℓ_p regularizer.

RESULTS  for $p = 2$

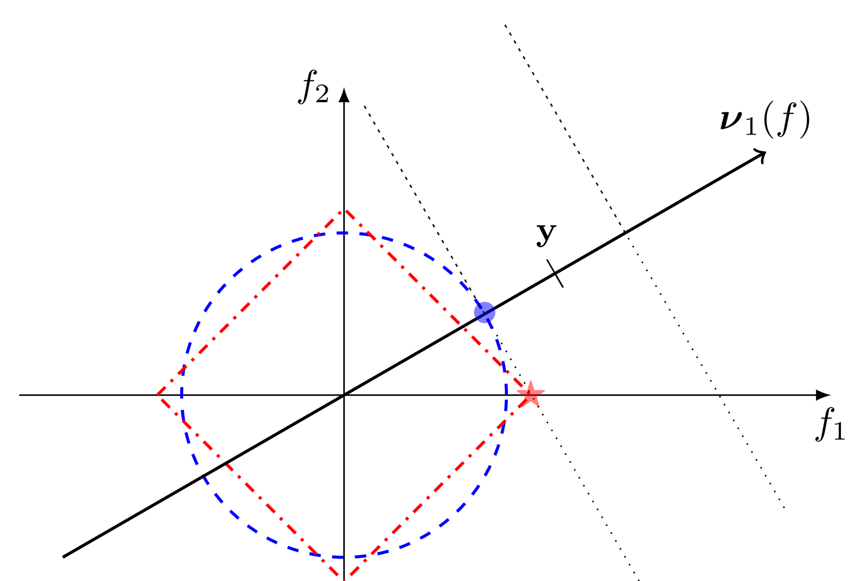
- For Tikhonov-regularized least-squares (Lipschitz stability, *i.e.*, $\beta = 1$)

$$\|f_{\mathbf{y}_1} - f_{\mathbf{y}_2}\|_{\mathcal{H}} \leq \max_{m \in \{1,2,\ldots,M\}} \frac{\sqrt{\sigma_m}}{\sigma_m + 2\lambda} \|\mathbf{y}_1 - \mathbf{y}_2\|_2$$

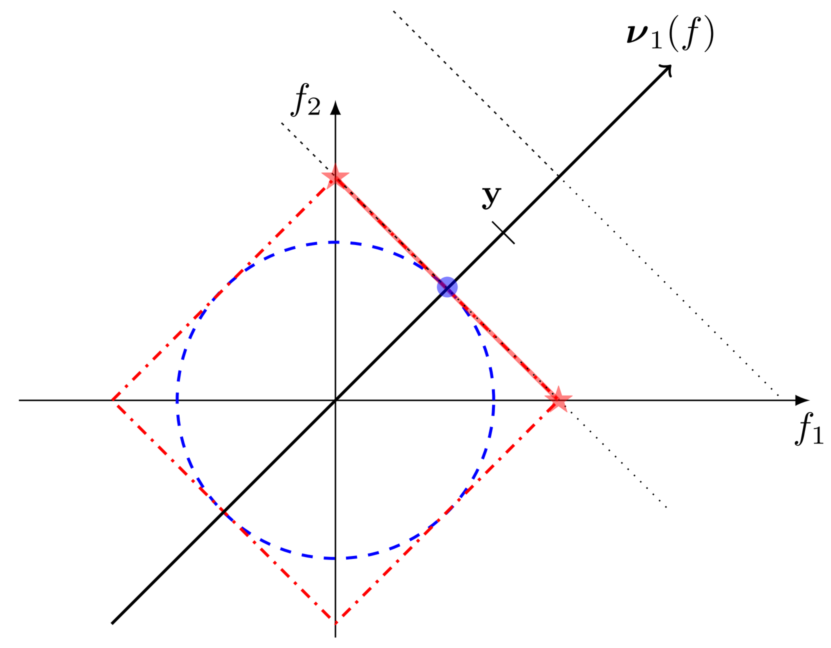
σ_m: eigenvalues of the Gram matrix $\mathbf{H}_{m,n} = \langle \nu_n, \nu_m \rangle$



RESULTS  for $p = 1$



- Unique, sparse solution
- Lipschitz stability, *i.e.*, $\beta = 1$, but unknown K.



- Infinite set of solutions, sparse extremes
- Stability not even defined

RESULTS for other p

- For $p \in (1,2)$, local Lipschitz stability

$$\|f_{\mathbf{y}_1} - f_{\mathbf{y}_2}\|_{L_p} \leq \frac{(2\,r_p(Y))^{2-p} K_p}{\lambda p(p-1)} \|\mathbf{y}_1 - \mathbf{y}_2\|_2$$

- For $p \in (2, \infty)$, Hölder stability

$$\|f_{\mathbf{y}_1} - f_{\mathbf{y}_2}\|_{L_p} \leq \left(\frac{2^{p-2} K_p}{\lambda p}\right)^{\frac{1}{p-1}} \|\mathbf{y}_1 - \mathbf{y}_2\|_2^{\frac{1}{p-1}}$$

CONCLUSION

- Traditional image-reconstruction methods do provide concrete stability guarantees, if properly used.
- Improved insight into the workings of ℓ_p-regularized linear inverse problems

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